Upscaling of Hydraulic Properties of Fractured Porous Media: Full Permeability Tensor and Continuum Scale Simulations


Abstract

The simulation of flow and transport phenomena in fractured media is a challenging problem. Despite existing advances in computer capabilities, the fact that fractures can occur over a wide range of scales within porous media compromises the development of detailed flow simulations. Current discrete approaches are limited to systems that contain a small number of fractures. Alternatively, continuum approaches require the input of effective parameters that must be obtained as accurately as possible, based on the actual fracture network or its statistical description.

In this work, a novel method based on the utilization of the Delta-Y transformation is introduced for obtaining the effective permeability tensor of a 2D fracture network. This approach entails a detailed description of the fracture network, where each fracture is represented as a segment with a given length, orientation and permeability value. A fine rectangular grid is then superimposed on the network, and the fractures are discretized so that each one of them is represented as a connected sequence of bonds on the grid with a hydraulic conductivity proportional to the ratio of effective permeability over fracture discretization length. The next step consists of the selection of a coarser rectangular grid on which the continuum simulation is performed. In order to obtain the permeability tensor for each one of the resulting blocks, the Delta-Y method is used.

Finally, the resulting continuum permeability tensor is used to simulate the steady-state flow problem, and the results are compared with the actual flow pattern yielded by the fracture network simulation. The results obtained with both methods follow a similar flux pattern across the reservoir system. This shows that the proposed approach allows for efficient perform upscaling of hydraulic properties by honoring both the underlying physics and details of fracture network connectivity.

1. Introduction

The implementation of successful oil and gas exploitation plans in naturally fractured reservoirs, relies on the ability to predict the physical processes that take place in the subsurface. A way to perform these predictions is through detailed and robust numerical models that can capture the geometrical complexity of individual fractures, the fracture/fracture and the fracture/porous media interaction, and the underlying physical and chemical processes.

The complexity of fractured geologies has limited the predictive ability of current models. However, with the advent of new computer capabilities, as well as, new numerical techniques and physical models, the implementation of increasingly reliable simulations of flow and reactive transport in complex fractured geologies is becoming a possibility.

A popular approach to modeling flow and transport in fracture systems is to consider the effect of each individual fracture on the simulation. The advantage of this approach is that the geometric features of the fractures can be incorporated in a very detailed fashion. The main disadvantage is the high computational cost involved and the fact that the fracture/porous medium interaction is difficult to incorporate.

To deal with the geological complexities, flow and transport calculations in fractured porous media rely on simplifying assumptions in terms of fracture geometry and fracture/porous media interaction. For example, two dimensional models have been used under the assumption that individual fractures are vertical and big enough to cross the entire unit in the vertical direction. Full three-dimensional simulators have been developed [1-3], however, due to the complexity of simulations, combined with the lack of data required to perform them, two-dimensional models remain popular.

Flow simulations in fractured porous media are generally carried out using continuum methods. According to those types of models, transport properties of the fracture network is modeled using continuum transport coefficient. When the porous medium is permeable, fractures and porous media are assumed to coexist in the same spatial coordinates while their interaction is incorporated by adding an exchange term to the equations. This kind of schemes are widely known as dual porosity models and have been shown to be valid for especial situations [4].
Fracture network modeling was first introduced by Long et al. [5] with the purpose of computing a permeability tensor for continuum simulations. They used a fracture network model where fractures were represented by lines randomly located in a two-dimensional space. The orientation of the fractures was chosen by sampling a given orientation distribution. They computed the conditions for homogenization based on the fact that the quantity $1/\sqrt{k(\theta)}$ may be fitted by an ellipse in polar coordinates.

Long et al. [5] introduced the concept of equivalent porous media for modeling flow in fractured geologies. They suggested that a fractured rock could be represented by a single permeability tensor so that the conventional continuum Darcy’s law equation could be implemented. It has been pointed out recently [6] that a single tensor is not sufficient to capture the complexity of the flow in the fracture network.

In this paper we introduce a homogenization approach to obtain a porous media equivalent to the fracture network. In order to improve the continuum representation, multiple permeability tensors are defined. Also, we implement an efficient Delta-Y [7] transformation in order to avoid the solution of the flow equations. We used the permeability field thus obtained to compute the pressures by using the Discontinuous Galerkin [8] method which matched reasonably well with the solutions obtained by solving the discrete network.

This paper is organized as follows: In section 2 we present the fracture network model used; section 3 deals with the fluid flow calculations in the discrete network; in section 4 we explain the method employed for the calculation of the effective permeability; in section 5 we compare results from continuum and discrete simulations; finally, in section 6 we outline some conclusions.

2. Fracture Network Model

The fracture networks employed in this study are created by generating randomly located lines within a rectangular domain. Orientation is randomly chosen within $(-\alpha + \beta, \alpha + \beta)$. Fracture length and aperture may be, in general, sampled from a probabilistic density function, although, for the examples presented here, it is assumed that all fractures have the same length and aperture.

Two examples of such networks are presented in Figure 1 and 2. In both of these samples all fractures have the same length $l = 0.05$ in some arbitrary length units and have been generated over a square region of side $\sqrt{2}$. The square regions at the center of Figures 1 and 2 represent the area where the fluid flow will be computed and have a side length equal to 1.

Figure 1. Fracture network consisting of 6400 lines of length $l = 0.05$ over a square generation domain of linear size equal to $\sqrt{2}$. The blue rectangle at the center represents the domain for flow calculations.

Figure 2. Fracture network with a line density equal to $N = dl^2 = 13.5$. The blue square represents a $1\times1$ region used for flow simulation.
The network depicted in Figure 1 contains a total of 6400 randomly oriented lines. Robinson [9] defines a dimensionless density according to

\[ N = \frac{d}{l^2}, \]

where \( d \) is the fracture areal density while \( l \) is the fracture length. By performing Monte Carlo simulations, Robinson is able to show that, on average, the onset of percolation for this kind of system will occur when \( N \) reaches the critical value \( N_c \approx 1.45 \). According to equation (1) the system shown in Figure 1 has a dimensionless density \( N = 8.0 \).

As a second example, in Figure 2 we show a network containing 10800 lines, randomly oriented in the range defined by \((\pi/2, \pi)\). In this case the dimensionless density \( N = 13.5 \). Robinson shows that when the range of orientation is restricted the onset of percolation occurs at a higher density defined, approximately, by a factor of \( \alpha = \pi/\Delta \), where \( \Delta \) is the angle range along which the fractures can be oriented. In the case considered here \( \Delta = \pi/2 \) so that \( \alpha = 2 \), this indicates that both networks are well connected.

### 3. Fluid Flow Calculations

In order to compute the flow equations in a fracture network it is necessary to solve the mass balance equations at the nodes (intersection between two lines). To perform this task, it is assumed that the local flow in the individual fracture segments is given by Darcy’s law, i.e.,

\[ q_{ij} = g_{ij} (p_i - p_j), \]

where \( q_{ij} \) is the flow between nodes \( i \) and \( j \), \( p_i \) and \( p_j \) are the pressures at the respective nodes, and \( g_{ij} \) is the hydraulic conductance of the segment given by

\[ g_{ij} = A \frac{k}{\mu \Delta x_{ij}}, \]

where \( A \) is the cross section of the fracture, \( \mu \) is the fluid viscosity, \( \Delta x_{ij} \) is the distance between the nodes and \( k \) is the fracture permeability. Furthermore, if Poiseuille’s law [10] is assumed valid, then the permeability is related to the fracture aperture.

The pressures at the line intersections are given by the following linear set of equations

\[ \sum g_{ij} (p_i - p_j) = 0, \]

which must be solved with prescribed boundary conditions.

Figure 3 and 4 show the results for the flow in the fracture networks depicted in Figure 1 and 2, respectively, for a constant head boundary condition in the horizontal direction. In these figures, the flux over each segment is represented by a yellow arrow following the direction of the pressure drop, with length proportional to the magnitude of the flux. By comparing these figures it is evident that the flow direction is conditioned by the fracture orientation.

A close look at these results establishes the existence of no flow regions and very active regions. Also, as it will become apparent later, the anisotropic behavior is, in general, scale-dependent as the flow direction is different at spatial...
locations. These effects are associated with connectivity fluctuations that occur naturally in this type of systems.

The solution of the set of equations (4) provides values for the fluid pressure at the fracture intersections. In order to represent this solution, and for further comparison with the continuum solution, the pressures are averaged over a $32 \times 32$ grid. The results corresponding to the fracture network shown in Figure 2 are presented in Figure 5. Notice that the pressure profile is consistent with the flow pattern observed in Figure 4.

![Figure 5. Solution for the pressure at the fracture intersections homogenized using a 32X32 square grid.](image)

4. Porous Media Equivalents

Solving the mass balance equations in the fracture network is a computationally expensive task. Generally, it is not feasible to perform this kind of computation at the scales required for reservoir or ground water simulations. In order to avoid the detailed solution of the mass balance equations in the fracture network, Long et al. [5] suggested the idea of porous media equivalents to avoid this limitation. Their method consisted of calculating a full permeability tensor through the solution of the flow equations in several different directions.

It has been pointed out that a single tensor may be insufficient to account for the complexity of the flow in the fractures. In order to improve the predictive capabilities of the porous medium equivalent, we suggest the use of the permeability field, in a way that the local connectivity fluctuations may be realized through spatial variations of the transport properties.

The calculation of the individual tensors is a key aspect of porous media equivalent methods. It is required that the procedure employed for this computation be as inexpensive as possible in terms of computer CPU time.

In order to efficiently compute the permeability tensor for the different subdomains we implemented the following procedure:

1. Choose an upsampling grid so that the fractured region is divided into different subdomains.
2. Isolate the fractures that intersect the considered subdomain.
3. Discretize the subset of fractures over a fine rectangular grid and map the conductances onto the corresponding links (see Figure 6).

![Figure 6. Local discretization for the application of the Delta-Y algorithm.](image)

4. Apply local boundary conditions (constant head from left to right, for instance).
5. Use the Delta-Y transformation method in order to calculate the permeability along two orthogonal directions.
6. Repeat steps 1-5 in different directions until a prescribed number of data points $(\theta, k)$ are obtained.
7. Fit an ellipse to the function $k(\theta)$. This ellipse represents the permeability tensor (more accurately fit an ellipse to the function $f(\theta) = 1/\sqrt{k(\theta)}$, though, fitting $k(\theta)$ is sufficient and more practical).
8. Use the continuum representation to calculate entries for the permeability tensor $k_{\theta\theta}$.
9. Repeat steps 1-8 for every subdomain to obtain a tensor representation for each subdomain.

Figure 7 shows the tensor permeability field for the fracture network shown in Figure 2, for $8 \times 8$, $4 \times 4$, $2 \times 2$ and $1 \times 1$ upsampling grids. These results suggest that the permeability tensor may be scale-dependent. These results also indicate that fluctuations in the permeability tensor decrease as the upsampling grid becomes coarser.
5. Discrete Solution vs. Continuum Solution

In order to compare the discrete solution with the continuum solution we calculated the pressure at the fracture intersections in the network. To do this, we overlaid a 32×32 square grid and calculated the average pressure inside each element. The results are shown in Figure 5.

The corresponding continuum solution was obtained using the Discontinuous Galerkin method using the 8x8 permeability tensor field depicted in Figure 7. The results from this simulation are shown in Figure 8.

A comparison of the pressure solutions represented in Figure 5 and 8 shows that the results obtained from the continuum simulation capture the general trend observed in the discrete solution.

It is possible to capture further details through the use of a more refined upscaling grid, however, one must be careful as homogenization conditions limit the size of the grid that can be used. These homogenization limits have been discussed by several authors, specially Bear [11]. The homogenization principle establishes that macroscopic parameters may be defined only after an REV (representative elementary volume) has been achieved; however, it is not clear what the effect in the continuum solutions will be if sub-REV transport parameters are employed for the simulations.

6. Summary and Conclusions

In this work we have presented the first steps to a systematic verification of the continuum approach to flow in fractured media. The method presented here can be summarized as follows:

1. The original fracture network is represented by an upscaling grid. A different permeability tensor is computed for each grid-block.
2. The method used to compute the effective permeability tensor does not rely on the solution of the flow equations but, rather, on the application of the Delta-Y transformation.
3. A direct comparison between the continuum and discrete results suggests that it is possible to obtain a reasonably accurate answer from continuum simulations as soon as the appropriate upscaling method is employed.

Nevertheless, several questions remain open in relation to the type of approach presented here, for example:

1. Upper and lower limits for the homogenization; effects that these limits have in the continuum simulations.
2. Extensions to more complex situations such as three-dimensional systems and the incorporation of fracture/matrix interaction.
3. Extensions of more complex physics such as transient phenomena and multiphase flow.

References


**Appendix: Delta-Y transformation method**

The Delta-Y transformation method for computing the effective conductivity of resistor networks was introduced by Frank and Lobb [7]. The idea was inspired by a transformation used in the area of electrical engineering according to which certain resistor connections could be modified while keeping the global behavior unchanged. Take for example, the transformation depicted in Figure A.1. The corresponding conductances may be expressed as follows:

\[
Y \rightarrow \nabla: \quad G_a = \frac{G_a G_1}{G_1 + G_2 + G_3}, \\
G_b = \frac{G_b G_1}{G_1 + G_2 + G_3}, \\
G_c = \frac{G_c G_2}{G_1 + G_2 + G_3}, \tag{A.1}
\]

and

\[
\nabla \rightarrow Y: \quad G_1 = G_a G_c \left[ \frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c} \right], \\
G_2 = G_c G_4 \left[ \frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c} \right], \\
G_3 = G_4 G_9 \left[ \frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c} \right]. \tag{A.2}
\]

Based on transformations (A.1) and (A.2) Frank and Lobb constructed a propagation algorithm by successive application of these equations. To illustrate the procedure consider Figure (A.1). The translation operation is repeated until the translated element “leaves” the system. The procedure is repeated until every element in the system except for those in the lower and right boundary. Then, the remaining elements are used to compute the macroscopic effective permeability in the chosen direction.