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Streamline Tracing on Unstructured Grids

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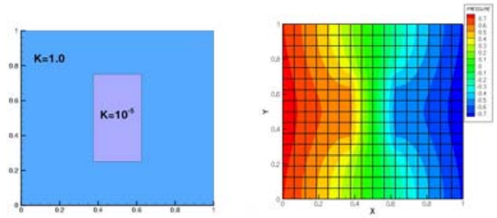
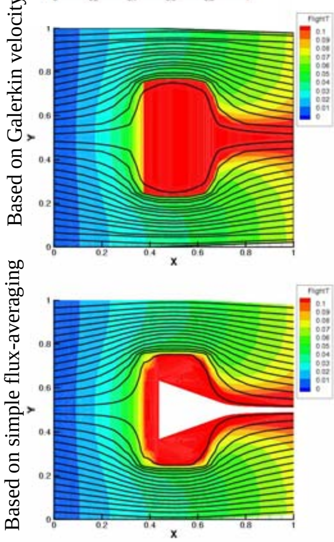
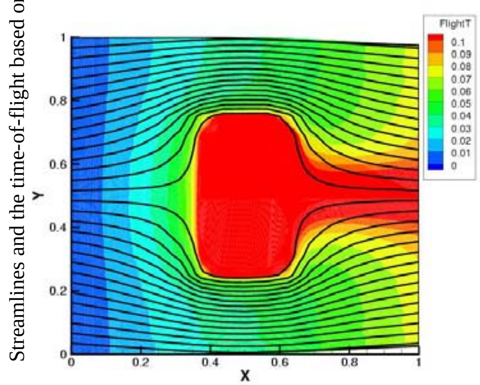
Single-phase flow with tracer injected on the left







Traditional methods for streamline tracing are mainly based on the Pollock Cartesian scheme and its extensions, which require dual grids to treat general tensor fields on unstructured meshes. Complexity and inflexibility of dual grids impose substantial limitations on the application of streamlines.

In this research, we propose a computationally efficient method to construct streamlines on the original grid, taking flow fields from arbitrary schemes including Galerkin finite element methods. An essential component of the proposed streamline construction is a fast processing of non-conservative velocity fields to recover or maintain the local conservation and normal flux continuity simultaneously, which involves balancing conservation residuals on each pair of adjacent elements using a Gauss-Seidel type iteration. Locally conservative velocities are extended from element faces to interiors using a local single-element mixed finite element method. Streamline and travel time are then obtained using standard methods of integration based on the Pollock algorithm.

The proposed approach is shown to possess several advantages: it treats general tensor fields on unstructured and even nonmatching grids, maintains the optimal order of accuracy for high order elements, and avoids the difficulties imposed by nonphysical sinks and sources from numerical inaccuracy. Several computational examples on structured and unstructured meshes are presented to demonstrate the effectiveness of the proposed method.

Gauss-Seidel (GS) Velocity Processing:

$$dT = \frac{d\alpha}{Q_1(\alpha)} = \frac{d\beta}{Q_2(\beta)} \qquad \tau = \phi \int J(\alpha(T), \beta(T))dT$$

1. Order interior faces (or interior edges for 2D).
2. Compute the local conservation residual for each element.
3. For each face, compute velocity correction by using the local residual balancing process between the elements sharing the face; update velocity data and local conservation residuals.
4. Check convergence: if true, go to step 5; otherwise, go to step 3.
5. Extend the velocity into element interiors:
   a) If only the zeroth-order compatibility is needed, interpolate to obtain the velocity defined over the entire domain;
   b) If the rth-order compatibility is desired, apply a local high-order mixed finite element method for each individual element using the above corrected flux as boundary conditions.

Quarter five-spot problem in heterogeneous media