Adaptive Discontinuous Galerkin Methods Applied to Reactive Transport in Porous Media

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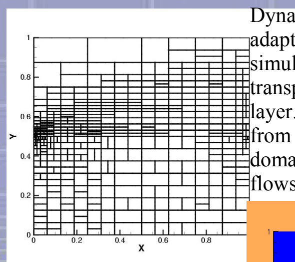
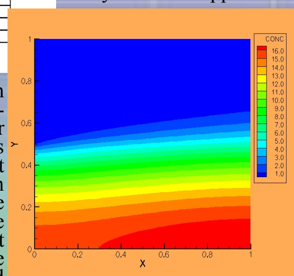
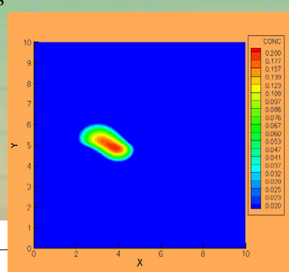
Numerous attractive properties of the Discontinuous Galerkin (DG) methods have given promise to reliable and accurate solutions of reactive transport problems in porous media. The flexibility of DG allows for general non-conforming meshes with variable degree of approximation, which leads to efficient and effective adaptivity. DG is locally mass conservative, and has less numerical diffusion than most conventional algorithms. DG handles rough coefficient problems and captures the discontinuity in the solution very well. Moreover, with appropriate meshing, DG is capable of delivering exponential rates of convergence. For time-dependent problems in particular, the mass matrices are block diagonal for DG, but not for conforming methods. This provides a substantial computational advantage, especially if explicit time integrations are used.

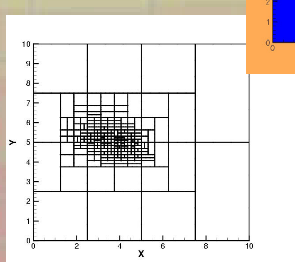


Dynamically and anisotropically adaptive discontinuous Galerkin simulation of a contaminant transport involving a boundary layer. Contaminant is released from the lower half of the domain while groundwater flows only in the upper half.



The coupling of a diffusion process and an advection-dominated process over two adjacent subdomains results in a time-dependent thin concentration boundary layer on the interface. It should be observed that the aspect ratios of elements are driven by the physics and the start region of boundary layer is intensively refined.





Simulation of a contaminant transport into two layers of rock with heterogeneous sorption. Plots are a contaminant concentration contour and the mesh using a discontinuous Galerkin method powered by dynamic adaptivities. Contaminant advects slower in bottom layer than in upper layer due to retardation effect.

A posteriori error estimate in $L^2(H^1)$ norm for primal DGs

$$\left\|\sqrt{\phi}\left(C^{DG}-c\right)\right\|_{L^\infty(L^2)}+\left\|\mathbf{D}^{1/2}(\mathbf{u})\nabla\left(C^{DG}-c\right)\right\|_{L^2(L^2)}\le K\left(\sum_{E\in\mathbf{E}_h}\eta_E^2\right)^{1/2}$$

$$\eta_E^2=h_E^2\left\|R_I\right\|^2_{L^2(L^2(E))}+\sum_{\gamma\in\partial E\cap\partial\Omega}h_\gamma\left\|R_{B1}\right\|^2_{L^2(L^2(\gamma))}$$

$$\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}h_\gamma\left\|R_{B1}\right\|^2_{L^2(L^2(\gamma))}+\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}\frac{1}{h_\gamma}\left\|R_{B0}\right\|^2_{L^2(L^2(\gamma))}$$

$$\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}h_\gamma\left\|R_{B0}\right\|^2_{L^\infty(L^2(\gamma))}+\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}h_\gamma\left\|\partial R_{B0}/\partial t\right\|^2_{L^2(L^2(\gamma))}$$

A posteriori error estimate in $L^2(L^2)$ for SIPG

$$\left\|C^{DG}-c\right\|_{L^2(L^2)}\le K\left(\sum_{E\in\mathbf{E}_h}\eta_E^2\right)^{1/2}$$

$$\eta_E^2=\frac{h_E^4}{r^4}\left\|R_I\right\|^2_{L^2(L^2(E))}+\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}\left(\frac{h_\gamma}{r}+\delta r h_\gamma\right)\left\|R_{B0}\right\|^2_{L^2(L^2(\gamma))}$$

$$+\frac{1}{2}\sum_{\gamma\in\partial E-\partial\Omega}\left(\frac{h_\gamma^3}{r^3}\left\|R_{B1}\right\|^2_{L^2(L^2(\gamma))}\right)+\sum_{\gamma\in\partial E\cap\partial\Omega}\left(\frac{h_\gamma^3}{r^3}\left\|R_{B1}\right\|^2_{L^2(L^2(\gamma))}\right)$$

A discontinuous Galerkin solution of the Andra-Couplex1 benchmark case. Plots display the Iodine-129 concentration profile and the mesh structure at 2 million years of simulation time. It was observed in this test case that effective mesh adaptations eliminate or reduce the need of slope limiters, especially in later simulation times.