Modelling and analysis of newsvendor-based trading options in supply chains

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Abstract: Options are introduced into supply chain management to improve the capability of handling demand uncertainty and hence seek better performance of the participants. An option model based on the newsvendor problem is presented to quantify and price a trading contract in a supply chain. With trading options, buyers (or retailers) can either order products from suppliers or purchase options from other retailers, and decide whether to buy or sell their remaining options in the second period after demand is realised in the first period. This paper examines how trading options work in a supply chain consisting of one supplier and a set of retailers in both competitive and cooperative scenarios. Using the concept of best response in game theory, the outcomes of option trading with interdependent demands are analysed. Depending on the current inventory, options in hand and demand information of the second period, the optimal trading quantity in the non-interdependent demand model could be found, where trading quantity is irrelevant to options price.

Keywords: supply chain management; options; newsvendor model; game theory.


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1 Introduction and literature review

Options are introduced into supply chain management to improve the capability of handling demand uncertainty and hence seek better performance of the participants (van Mieghem, 1999; Cachon, 2002). It has been shown that options (Barnes-Schuster et al., 2002) or options-like contract such as buy-back contract (Pasternack, 1985; Emmons and Gilbert, 1998), backup contract (Eppen and Iyer, 1997) and quantity-flexibility contract (Tsay and Lovejoy, 1999) can provide both suppliers and retailers with flexibility thus improve supply chain performance. Chen et al. (2004) described a competitive newsvendor problem using price as the competing strategy. They developed a fairly general demand model that does not depend on a specific functional form, and it captures the differentiable and substitutable features of products using price elasticity. Chen and Parlar (2007) considered an extension of the single-period inventory model with stochastic demand where a put option can be purchased to reduce losses resulting from low demand. They showed that the newsvendor’s expected profit with the option is independent of the strike price and strike quantity, but the option has the effect of reducing the newsvendor’s expected profit by the amount of risk premium paid to the option writer. Cheng et al. (2006) studied a situation with a general option-future contract, which generalised several widely practiced contracting schemes such as capacity reservation and buy-back/return policies. Based on the Stackelberg model, they derived the optimal ordering decision and pricing decision with respect to manufacturers and suppliers, they also found a negotiation mechanism for supply chain coordination over the no-flexibility contract. In this paper, the options refer to the retailers’ rights to order products from suppliers and then sell them to end-users during the selling season.
By allowing the real options to be tradable between retailers, they are able to pool the risks associated with demand uncertainty (Shi et al., 2004). Unlike financial options, the price of the trading options in this study is the result of the negotiation process between a buyer and a seller. It has been proposed that one way to price the options is to use the equilibrium market price (Shi et al., 2004). However, the market price for the options cannot be determined directly, because proprietary information is not shared among all retailers. Thus, a motivation is to see how the trading options’ pricing makes the trading negotiation more flexible and brings more profits for all the participants in the supply chain. In this paper, the previous model is extended by changing the options trading price from a pre-determined value to a decision variable that should be determined along with the trading quantity.

Most related literatures use the independent identity distribution to describe demands, while in a more realistic environment different retailers in the same area/district may share the market demands. Another contribution of this paper is to extend the previous model to a demand-interdependent scenario where unsatisfied customers may switch to other retailers for acquiring products. Taking account of the correlation of retailers’ demand distributions, this paper investigates a good strategy for quantifying and pricing the trading options among retailers.

In the single-period model, we consider a newsvendor model consisting of one supplier and two retailers, where the supply contract follows call options. At the beginning of the period, the retailer orders a certain number of products from its supplier, at the same time, purchases a number of units of options from the supplier. In this paper, options are embedded in the supply chain contract and can thus be traded in an open market. A general game theoretic framework is adopted for studying the trading quantity and trading price in two scenarios: competitive and cooperative, which are applicable in different situations. In the competitive scenario, the player reacts to other players’ decisions and makes its best responses by finding the equilibrium on the intersection of two best response functions. In the cooperative scenario, the players try to maximise their joint profits and split the joint profits in a pre-determined scheme. Here, optimal trading quantity is irrelevant to the option price.

The rest of the paper is organised as follows. A problem description and formulation is presented in Section 2. Section 3 presents the analysis of options trading strategy for different options-holders: buyer and seller. Section 4 provides solutions for options trading negotiation in both cooperative and non-cooperative scenarios. The paper is concluded in Section 5 with a summary and suggestions for further research.

2 Problem formulation and preliminary results

The single-period stylised newsvendor model is extended by embedding call options into it and incorporating trading options between retailers. The supply chain studied is composed of two parties: a supplier producing short-life-cycle products, and a set of retailers \{1,2,\ldots,n\} who order products from the supplier. This paper focuses on the each retailer’s independent decisions on how many units of products and real options to purchase at time 0 to cover a selling season \([0,\tau]\). Because of the long lead-time, retailers have no opportunity to replenish their inventory through new orders once the season begins. However, retailers are allowed to adjust their inventory positions by trading options with each other at a certain time \((0 < t < \tau)\) during the season.
At time $t = 0$, retailers can obtain products from the supplier by two ways: either through ordering products or by buying and exercising call options. At $t = 0$, $Q_i$ units of products are physically delivered to retailer $i$. Suppose retailer $i$ purchases $q_i$ options, each unit of options gives retailer $i$ the right to buy one unit of product at strike price $\nu$ during the selling season $[0, \tau]$. Retailer $i$ only exercises options when $D_i > Q_i$. When $Q_i$ is insufficient to meet all demands, the retailer will exercise up to $q_i$ options if trading options is not allowed. During $0 < t < \tau$, a market is open to retailers where they can trade their options.

2.1 Notation and assumptions

The notations in Table 1 will be used for all the models throughout the paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i$</td>
<td>Customer demand per period, supplied by the retailer $i$</td>
</tr>
<tr>
<td>$F$</td>
<td>Cumulative distribution function (c.d.f.)</td>
</tr>
<tr>
<td>$f$</td>
<td>Probability distribution function (p.d.f.)</td>
</tr>
<tr>
<td>$r$</td>
<td>Retailer’s unit selling price</td>
</tr>
<tr>
<td>$m$</td>
<td>Supplier’s unit cost</td>
</tr>
<tr>
<td>$w$</td>
<td>Supplier’s unit wholesale price</td>
</tr>
<tr>
<td>$s$</td>
<td>Unit salvage value</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>Order quantity for each retailer $i$ at time $t = 0$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Retailer $i$’s options purchasing quantity at unit cost</td>
</tr>
<tr>
<td>$c$</td>
<td>Unit options price at $t = 0$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Strike/exercise price</td>
</tr>
</tbody>
</table>

2.2 The retailer’s order decisions (at time 0 without options trading)

At time $0$, retailer $i$ ($I = 1, 2, \ldots, n$) decides to order $Q_i$ units of products and purchase $q_i$ units of call options. The retailer always first fulfills the customer demand by using its on-hand inventory $Q_i$. When the demand exceeds $Q_i$, the retailer will turn to exercise up to $q_i$ options.

The expected payoff of retailer $i$ is given as follows:

$$E\pi_i(Q_i, q_i) = E_{\nu} \left[ r \min(D_i, (Q_i + q_i)) ight]$$

\[+ s(Q_i - D_i) - wQ_i \]

\[- cq_i - \nu \min(q_i, (D_i - Q_i)) \]

The first term represents the total revenue from the retailer’s sales to customers, which is constrained by demand and supply. The second term is the salvage value of the unsold products left in the retailer. The third term refers to the cost of ordering products corresponding to unit wholesale price. The fourth and fifth terms represent the cost of purchasing call options and exercising required options, respectively.
Consider a single-period, single-product model involving a single supplier and a few retailers. At the beginning of the selling season, each retailer \( i \) places an order of \( Q_i \) units of products from the supplier, paying a price of \( w \) for each unit of product. The retailer can also purchase \( q_i \) units of call options from the supplier at a price of \( c \) per unit. Each option gives the retailer the right but not the obligation to receive an additional unit of product on a certain time in the future immediately when it exercises the options, at a cost of \( \nu \) per unit.

This call option increases flexibility to supply contracts, allowing buyers to order additional units of products at a premium (corresponding to option exercise price), when initial order quantity is insufficient to meet all demands.

To allow sufficient model generality, two scenarios are discussed in supply chain analysis:

1. competitive newsvendor model with interdependent demand
2. cooperative newsvendor model with non-interdependent demand.

### 3 Newsvendor-based option model

#### 3.1 Competitive scenario with interdependent demands

First, we consider using game theory of the classic newsvendor problem with options, where two retailers are competing on product availability.

**3.1.1 Ordering strategy at time zero**

In a competitive game, consider that the retailers’ payoffs are interdependent by assuming the two retailers sell the same products (ordering from the same single supplier). If retailer \( i \) is out of stock, all unsatisfied customers will switch to retailer \( j \).

As a result, retailer \( i \)’s total demand becomes:

\[
D_i = (D_i - Q_i + q_i)
\]

That is, the summation of retailer \( i \)’s own demand and the demand transferred from unsatisfied customers by retailer \( j \).

Let \( T_i = Q_i + q_i \), then the expected payoff function of retailer \( i \) can be rewritten as:

\[
E\pi_i(Q, T) = (v - w + c)Q_i - (v - y)\int F_{D_i,D_j,Q_i,q_j}(D)\,dD + (v - c)T_i - (v - y)\int F_{D_i,D_j,Q_i,q_j}(D)\,dD
\]

The best response functions can be found by optimising each retailer’s payoff functions w.r.t. the player’s own decision variable \( T_i \) while taking the competitor’s strategy \( T_j \) as the given input. The results are:

\[
T_i^* (T_j) = F^{-1}_{D_i,D_j,Q_i,q_j}\left(\frac{y - c - v}{v - y}\right)
\]

\[
T_j^* (T_i) = F^{-1}_{D_i,D_j,Q_i,q_j}\left(\frac{y - c - v}{v - y}\right)
\]
In this two-player game, given the decision of retailer \( j \), the response is the best one that retailer \( i \) could expect. Clearly, an outcome in which both retailers choose their best responses (strategy) is a candidate for the non-cooperative solution. Such an outcome is a Nash equilibrium of the game (Asha and Benkatraman, 1994; Dagan et al., 2002).

It is helpful to analyse the two-player game model because it can provide some managerial insights. Some insights can be obtained by finding out how each retailer reacts to the decisions made by the other retailer and computing the first order indifference of the function:

\[
\frac{\partial T_i^i(T_j)}{\partial T_j} = \left( \frac{\partial^2 \pi_i}{\partial T_i \partial T_j} \right) \frac{\partial^2 \pi_i}{\partial T_j^2} < 0
\]  

This expression indicates that each retailer’s best response is strictly decreasing with the other retailer’s decision. The intersection of the two best responses is a fixed point that exactly represents the Nash equilibrium location.

It is more appealing that Nash equilibrium is a necessary condition for the prediction of any rational behaviour by retailers who always choose to maximise their own payoff given the competitor’s strategy.

Once obtaining the optimal values for retailers (i.e. \( T_i^* \) and \( T_j^* \)), the optimal order quantity \( Q_i^* \) can be computed by letting \( \partial \pi_i(Q_i, T_j^*) / \partial Q_i = 0 \) and then the optimal options quantity \( q_i^* \) (same expression for \( j \)) can be obtained. The results are given as follows:

\[
Q_i^* = F^{-1}_{D_i + (D_j - T_j^*)} \left( \frac{r - c - w}{r - s} \right), \quad Q_j^* = F^{-1}_{D_j + (D_i - T_i^*)} \left( \frac{r - c - w}{r - s} \right)
\]

\[
q_i^* = T_i^* - Q_i^*, \quad q_j^* = T_j^* - Q_j^*
\]

### 3.1.2 Trading options at time \( t \)

At time \( t \), the decisions faced by the retailers can be defined along three dimensions: buying or selling options, the price (denoted as \( \tilde{c} \)) of buying or selling options, and the quantity (denoted as \( \Delta q \)) of buying or selling options. The demands across retailers are assumed to be independently and identically distributed.

Some notations are as follows:

- \( D_i^1 \) and \( D_i^2 \) denote the random demands faced by retailer \( i \) in the first period \([0,t]\) and the second period \([t, \tau]\), respectively.
- \( d_i^t \) denotes the actual demand realised by retailer \( i \) in the first period, \( \{d_i^t : i = 1, 2, \ldots, n\} \) are different.
- \( \tilde{D}_i = D_i^2 | d_i^1 \) is the conditional distribution of the demand in the second period.
- \( \tilde{Q}_i \) denotes the quantity of products retailer \( i \) holds in inventory at time \( t \), \( \tilde{Q}_i = (Q_i^* - d_i^t) \).
- \( \tilde{q}_i \) denotes the quantity of options retailer \( i \) holds in inventory at time \( t \), \( \tilde{q}_i = (q_i^* - (d_i^t - Q_i^*)) \).
Let \( F_{i,j}^{\prime}(\tilde{D}_i, \hat{q}_i, \hat{q}_j + \Delta q_j) \) denote the cdf of retailer \( i \)'s (buyer) random demand distribution \( \tilde{D}_i \). Hereafter, simplify it to \( F_{i,j}^{\prime} \) and \( \{ F_{i,j}^{\prime} : i = 1, 2, \ldots, n \} \) are no longer identical. Similarly, let \( F_{j,i}^{\prime}(\tilde{D}_j, \hat{q}_j, \hat{q}_i + \Delta q_i) \) denote the cdf of retailer \( j \)'s (seller) random demand distribution \( \tilde{D}_j \).

Then buyer’s expected surplus payoff function is given by:

\[
\Delta E\pi_i(\hat{c}, \Delta q) = E_{\hat{c}} \left[ (r - v) \min(\Delta q, (\tilde{D}_i - \hat{q}_i, \hat{c})) - \hat{c} \Delta q \right] \\
- \hat{c} \Delta q + (r - v) \Delta q - (r - v) \int_{\hat{q}_i}^{\hat{q}_i + \Delta q} F_{j,i}^{\prime}(\tilde{D}_j) d\tilde{D}_j
\]

(9)

And the seller’s expected surplus payoff function is as follows:

\[
\Delta E\pi_j(\hat{c}, \Delta q) = E_{\hat{c}} \left[ (r - v) \min(\Delta q, (\tilde{D}_j - \hat{q}_j, \hat{c})) - \hat{c} \Delta q \right] \\
- \hat{c} \Delta q + (r - v) \Delta q - (r - v) \int_{\hat{q}_j}^{\hat{q}_j + \Delta q} F_{j,i}^{\prime}(\tilde{D}_i) d\tilde{D}_i
\]

(10)

When \( \Delta q = 0 \), \( \Delta E\pi_i(\hat{c}, \Delta q) = \Delta E\pi_j(\hat{c}, 0) = 0 \) indicates that there is no surplus profit if there is no any option trading. The expected surplus function \( \Delta E\pi_i(\hat{c}, \Delta q) \) is concave in \( \Delta q \), hence, taking derivative w.r.t. \( \Delta q \) on the objective function in equation (9) and letting it be zero, we can find the solution that yields the optimal \( \Delta q \) with a certain \( \hat{c} \) value.

In the second period, the two retailers still face with interdependent demands and the realised demands in the first period become known.

\[
F_{i,j}^{\prime}(\hat{q}_i + \hat{q}_j + \Delta q) = \frac{(r - \hat{c} - v)}{(r - v)}
\]

(11)

Thus the buyer’s best response (function) to the seller’s strategy \( \Delta q_j \) is given as:

\[
\Delta q_i^* (\Delta q_j) = F_{i,j}^{\prime}(\hat{q}_i + \hat{q}_j + \Delta q) = \frac{(r - \hat{c} - v)}{(r - v)}
\]

(12)

Now the best response functions can be used to find the Nash equilibrium of the trading option game. When analysing games with a seller and a buyer, some intuition can be gained by finding out how the buyer reacts to an increase in the trading quantity offered by the seller. Taking implicit differentiation as follows:

\[
\frac{\partial \Delta q_i^* (\Delta q_j)}{\partial \Delta q_j} = \frac{(r - v) f_{i,j}^{\prime}(\tilde{D}_i, \hat{q}_i, \hat{q}_j + \Delta q_j) Pr(D_i - \hat{q}_i + \Delta q_j > 0)}{(r - v) f_{i,j}^{\prime}(\tilde{D}_i, \hat{q}_i, \hat{q}_j + \Delta q_j) (\Delta q_j)} > 0.
\]

The expression indicates that the slope of the best response function is positive, which implies a result that the buyer’s best response is monotonically increasing with the seller’s strategy. Similarly, the seller’s best response function can be obtained given the buyer’s strategy on how many options to buy.

\[
F_{j,i}^{\prime}(\hat{q}_i + \hat{q}_j + \Delta q) = \frac{(r - \hat{c} - v)}{(r - v)}
\]

(13)
Newsvendor-based trading options in supply chains

\[ \Delta q_i^*(\Delta q_j^*) = \frac{F_{D_i+Q_i+\Delta q_i}^{-1}}{F_{D_i+Q_i+\Delta q_i}^{-1}} \left( \frac{(r-\tilde{c}-v)}{(r-v)} + \bar{Q}_j + \tilde{q}_j \right) \]  

(14)

\[
\frac{\partial \Delta q_i^*(\Delta q_j^*)}{\partial \Delta q_i} = \frac{(r-v)f_{D_i+Q_i+\Delta q_i}(\Delta q_i)}{(r-v)f_{D_i+Q_i+\Delta q_i}(\Delta q_i)} \Pr(D_i - \bar{Q}_i - \tilde{q}_j + \Delta q_i > 0) > 0
\]

However, these two best response functions result in an asymmetric game. The equilibrium is located on the intersection of the two best responses. Since both seller and buyer are rational and risk-neutral who make effort to achieve an agreement for each own maximum profit, the quantity of trading options should be equal such that \( \Delta q_i^*(\Delta q_j^*) = \Delta q_j^*(\Delta q_i^*) \), while the trading price \( \tilde{c} \) will be determined at the same time. In other words, both seller and buyer can reach an agreement for \( \Delta q \) through negotiating the trading price \( \tilde{c} \).

3.2 Cooperative model with non-interdependent demand

Consider a situation that is no longer classic newsvendor problem, the retailers are considered as distributors who are located separately and far away from each other, thus the demands on each location cannot be transferred to the others, that is, non-interdependent demands. Let \( F_{D_i} \) denote the c.d.f. of the random demand distribution \( iD_i \).

\{ \}

\( i \in \{1, 2, \ldots, n\} \) is not identical.

3.2.1 Buyer’s strategy

Firstly, consider the option buyer’s strategy. The buyer’s expected surplus profit after trading is given by:

\[
\Delta E \pi_x(\tilde{c}, \Delta q) = E_{D_i}(r-v)\min(\Delta q, (\bar{Q}_i - \tilde{q}_j)^+) - \tilde{c} \Delta q + (r-v)\Delta q - (r-v) \int_{\tilde{q}_j+\Delta q}^{\tilde{q}_j+\Delta q} F_{D_i}(\bar{Q}_i) d\bar{Q}_i
\]

(15)

Because no surplus profit will be gained if the buyer does not buy any options, we have \( \Delta E \pi_x(\tilde{c}, 0) = 0 \). Since the expected surplus function \( \Delta E \pi_x(\tilde{c}, \Delta q) \) is concave with respect to \( \Delta q \), this results in finding a unique optimal solution \( \Delta q \) with a certain \( \tilde{c} \) such that

\[
F_{D_i}(\tilde{Q}_i + \tilde{q}_j + \Delta q) = \frac{(r-\tilde{c}-v)}{(r-v)}
\]

(16)

Now we can obtain the benchmark \( \tilde{c}_i^* \) at which the retailer decides to buy or sell options, and the possible interval of \( \tilde{c} \) will also be given to achieve the trading agreement.

Let \( \tilde{c}_i^* \) satisfy \( F_{D_i}(\tilde{Q}_i + \tilde{q}_j) = \frac{(r-\tilde{c}_i^*-v)}{(r-v)} \), where the trading quantity \( \Delta q = 0 \).
Situation 1: $\check{c} > \check{c}_i$

When $\check{c} > \check{c}_i$, we have

$$\frac{\partial \Delta E\pi_n(\check{c}, \Delta q)}{\partial \Delta q} = (r-v)(F''_\check{q}(\check{Q} + \check{q}_r + \Delta q) - (r-v)F''_\check{q}(\check{Q} + \check{q}_r + \Delta q)) < 0$$

Thus, for any specific $\check{c} > \check{c}_i$, the $\Delta E\pi_n(\check{c}, \Delta q)$ is a decreasing function and we have $\Delta E\pi_n(\check{c}, \Delta q) < \Delta E\pi_n(\check{c}, 0)$. This leads to the following proposition:

Proposition 1: The buyer’s surplus profit is always negative if it buys options at a price $\check{c} > \check{c}_i$.

Situation 2: $\check{c} \leq \check{c}_i$

For a specific $\check{c} \leq \check{c}_i$, we can find the optimal options position $\Delta q^*$ that satisfies $\frac{\partial \Delta E\pi_n(\check{c}, \Delta q)}{\partial \Delta q} = 0$, and the following results can be obtained:

1. when $0 \leq \Delta q \leq \Delta q^*$, we have $0 = \Delta E\pi_n(\check{c}, 0) \leq \Delta E\pi_n(\check{c}, \Delta q) \leq \Delta E\pi_n(\check{c}, \Delta q^*)$.
2. when $\Delta q^* \leq \Delta q$, we have $\Delta E\pi_n(\check{c}, \Delta q) \leq \Delta E\pi_n(\check{c}, \Delta q^*)$.

Based on above results, the following proposition can be obtained:

Proposition 2: For a specific $\check{c} \leq \check{c}_i$, the expected surplus profit for a buyer is always non-negative when $0 \leq \Delta q \leq \Delta q^*$, where $\Delta E\pi_n(\check{c}, \Delta q) = \Delta E\pi_n(\check{c}, 0) = 0$, and its optimal trading quantity is $\Delta q^*$.

It can be seen that $\Delta E\pi_n$ is increasing with $\Delta q \in [0, \Delta q^*]$ and decreasing with $\Delta q \in [\Delta q^*, +\infty]$. (Note: $\Delta q^*$ is the upper bound of trading quantity that brings surplus profit to the buyer.)

In conclusion, the buyer’s strategy is to buy options at a price $\check{c} \leq \check{c}_i$, and when it buys options with quantity $\Delta q = \Delta q^*$, it can obtain the maximal expected surplus profit.

3.2.2 Seller’s strategy

Similar to buyer’s strategy, it can be proven that the seller’s strategy is to sell options at a price $\check{c} \geq \check{c}_j$, and when its selling option quantity becomes $\Delta q = \Delta q^*$, it has the maximal expected surplus profit. Having analysed the benchmark $\check{c}_j^*$ at which the retailer decides to buy or sell options, the following result can be obtained:

Proposition 3: At time $t$, the retailer’s strategy is either to buy options at a price $\check{c} \leq \check{c}_j^*$ or to sell options at a price $\check{c} \geq \check{c}_j^*$ to ensure that its expected surplus profit is nonnegative.
4 Trading price

Unlike financial options that are priced by using different techniques such as martingale approach (Hull, 2002), the pricing of such options in a market where the option trading occurs is the result of a private negotiation between two or more retailers. Usually, it is determined along with the trading quantity. Therefore, a trading is characterised by two parameters, namely trading price $\tilde{c}$ and trading quantity $\Delta q$. To simplify the analysis, a situation where there are only two retailers is considered. The superscripts $r_1$ and $r_2$ denote the retailer I and retailer II, respectively. Suppose $\tilde{c}_r^r < \tilde{c}_2^r$, according to Proposition 3, retailer I should sell (buy) its options at a price $\tilde{c} \geq (\geq) \tilde{c}_2^r$, and retailer II should buy (sell) options at a price $\tilde{c} \leq (\geq) \tilde{c}_1^r$. Therefore, the option trading takes place between the two retailers at a price $\tilde{c} \in [\tilde{c}_1^r, \tilde{c}_2^r]$. Retailer I is the seller and retailer II is the buyer. In the following, a game theoretic framework for studying such option trading in both cooperative and non-cooperative scenarios is presented.

4.1 Cooperative scenario

In the cooperative scenario, players are trying to maximise the joint expected surplus profit $\Delta E\pi_{\mu}(\tilde{c}, \Delta q)$. Here, subscript $\mu$ denotes the joint profit.

Define $\Delta E\pi_{\mu}(\tilde{c}, \Delta q) = \Delta E\pi_{\mu}^r(\tilde{c}, \Delta q) + \Delta E\pi_{\mu}^r(\tilde{c}, \Delta q)$, we have

$$
\Delta E\pi_{\mu}(\tilde{c}, \Delta q) = \tilde{c}\Delta q - (r-v)\Delta q + (r-v) \int_{\tilde{c}+\Delta q}^{\tilde{c}+\Delta q} F_{21}(\tilde{D}) d\tilde{D} - \tilde{c}\Delta q + (r-v)\Delta q - (r-v) \int_{\tilde{c}+\Delta q}^{\tilde{c}+\Delta q} F_{21}(\tilde{D}) d\tilde{D}
$$

$$
= (r-v) \int_{\tilde{c}+\Delta q}^{\tilde{c}+\Delta q} F_{21}(\tilde{D}) d\tilde{D} - \int_{\tilde{c}+\Delta q}^{\tilde{c}+\Delta q} F_{21}(\tilde{D}) d\tilde{D}
$$

(17)

Note that the joint profit is only a concave function of the trading quantity $\Delta q$, we can compute $\Delta q^*$ such that:

$$
\frac{\partial \Delta E\pi_{\mu}(\tilde{c}, \Delta q)}{\partial \Delta q} \bigg|_{\Delta q = \Delta q^*} = (r-v)(F_{21}(\tilde{Q}_1 + \tilde{q}_1 - \Delta q^*) - F_{21}(\tilde{Q}_1 + \tilde{q}_1 + \Delta q^*)) = 0
$$

The joint expected surplus profit $\Delta E\pi_{\mu}(\tilde{c}, \Delta q)$ has the maximum value at the trading point $\Delta q^*$. After establishing the equilibrium bargaining method for choosing an optimal trading quantity with a specific trading price $\tilde{c}$, the determination of the value of $\tilde{c} \in [\tilde{c}_1^r, \tilde{c}_2^r]$ will be discussed. In a cooperative scenario, a pre-determined scheme can be set up to split the joint expected surplus profit. The pricing of the trading options is, naturally, associated with the two players’ bargaining powers. For example, the seller could obtain $\theta \Delta E\pi_{\mu}(\tilde{c}, \Delta q^*)$ in profit and buyer will obtain $(1-\theta)\Delta E\pi_{\mu}(\tilde{c}, \Delta q^*)$ in profit, where $\theta \in (0,1)$ indicates that two players’ relative bargaining powers are defined as the proportions in which the retailers split the expected benefits of the joint expected surplus profit. In that case, the trading price $\tilde{c}$ can be obtained easily by solving

$$
\Delta E\pi_{\mu}(\tilde{c}, \Delta q^*) = \theta \Delta E\pi_{\mu}(\tilde{c}, \Delta q^*)
$$

(18)
From the above analysis, it can be seen that the joint expected surplus profit is only determined by the trading quantity $\Delta q_j$, the joint profit splitting is determined only by the trading price $\tilde{c}$.  

### 4.2 Non-cooperative scenario

In the non-cooperative scenario, retailers come across a random opponent in an open market where the option trading occurs at time $0 \leq t \leq \tau$. In this scenario, retailers only concern about their own expected profits. This situation can be modelled as a dynamic game where decisions are made over time. With different assumptions and settings, different solutions are provided to solve the dynamic game. Under some reasonable assumptions, the situation can be modelled as Rubinstein’s classic model (Binmore et al., 1986).

Specifically, in this dynamic game, each retailer in the game is assumed to be rational, risk-neutral and only concerned with its own interest. Further assuming that the information is symmetric, and each retailer will have a backup option (such as buying or selling options to a third retailer) if the bargain process breaks down. The expected surplus profits of backup options of retailer I and retailer II are $\tilde{\pi}_1$ and $\tilde{\pi}_2$, respectively. Time dependence is considered in the proposed model. With equal probability, one of the two retailers starts the game by proposing an offer with parameters $(\tilde{c}, \Delta q)$. The other retailers may either accept this offer in which case the negotiation ends or reject this offer and proposes a new offer to the opponent in which case the game restarts and enters next period. If the negotiation breaks down, then the two retailers will take their own corresponding backup options $\tilde{\pi}_1$ and $\tilde{\pi}_2$, respectively. Here, the discount factor is $\xi$.

When retailer I is proposing an offer, it tries to maximise

$$
\Delta E\pi_1(\tilde{c}, \Delta q) = \tilde{c}\Delta q - (r-v)\Delta q + (r-v) \int_{\tilde{\theta}_1}^{\tilde{\theta}_1+\Delta q} F_{\tilde{D}_1}(\tilde{D}_1)d\tilde{D}_1
$$

subject to

$$
\Delta E\pi_2(\tilde{c}, \Delta q) = -\tilde{c}\Delta q + (r-v)\Delta q - (r-v) \int_{\tilde{\theta}_2}^{\tilde{\theta}_2+\Delta q} F_{\tilde{D}_2}(\tilde{D}_2)d\tilde{D}_2 \geq \tilde{\pi}_2
$$

when the constraint is binding, we have

$$
\Delta E\pi_2(\tilde{c}, \Delta q) = -\tilde{c}\Delta q + (r-v)\Delta q - (r-v) \int_{\tilde{\theta}_2}^{\tilde{\theta}_2+\Delta q} F_{\tilde{D}_2}(\tilde{D}_2)d\tilde{D}_2 = \tilde{\pi}_2
$$

Thus, the retailer is trying to maximise:

$$
\Delta E\pi_1(\tilde{c}, \Delta q) = -\tilde{\pi}_2 - (r-v) \int_{\tilde{\theta}_2}^{\tilde{\theta}_2+\Delta q} F_{\tilde{D}_2}(\tilde{D}_2)d\tilde{D}_2 + (r-v) \int_{\tilde{\theta}_1}^{\tilde{\theta}_1+\Delta q} F_{\tilde{D}_1}(\tilde{D}_1)d\tilde{D}_1
$$

The optimal $\Delta q^*$ can be computed such that

$$
\frac{\partial \Delta E\pi_1(\tilde{c}, \Delta q)}{\partial \Delta q} \bigg|_{\Delta q^*} = 0.
$$
Thus, we have
\[
(r - v) \left( F_{2i}^{\ell_1} (\tilde{Q}_1 + \tilde{q}_1 - \Delta q') - F_{2i}^{\ell_2} (\tilde{Q}_1 + \tilde{q}_1 + \Delta q') \right) = 0
\]  
Expression (23) indicates the same result as that in the cooperative scenario can be obtained. Similarly, it can be shown that an optimal strategy for retailer II is to propose an offer with parameter \( \Delta q' \) satisfying equation (23). Since they have equal probability of proposing an offer, no matter who proposes the bargain first, the optimal trading quantity for each player is always the same as \( \Delta q' \) deduced above.

Proposition 4: The optimal joint trading quantity is also optimal trading quantity for both buyers and sellers.

It has been shown that both buyer and seller are proposing an offer with the same parameter \( \Delta q = \Delta q' \), there is no need for them to bargain about the trading quantity. The joint surplus expected profit is fixed for them when \( \Delta q = \Delta q' \). Therefore, the game now turns to be two players’ bargaining about the trading price \( \tilde{c} \), which determines how to split the joint expected surplus profit based on the above analysis.

The fixed expected joint surplus profit when an agreement is achieved is
\[
\Delta E \pi_{nj} (\tilde{c}, \Delta q') = (r - v) \left[ \frac{1}{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'} \int_{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'}^{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'} F_{2i}^{\ell_2} (\tilde{D}_1) d\tilde{D}_1 - \frac{1}{\tilde{Q}_1 + \tilde{q}_1} \int_{\tilde{Q}_1 + \tilde{q}_1}^{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'} F_{2i}^{\ell_1} (\tilde{D}_2) d\tilde{D}_2 \right]
\]  
and if negotiation breaks down each retailer will gain their own backup options’ expected surplus profits. They only accept the offer when the expected surplus profit is greater than their backup options’ expected surplus profit. Suppose that the backup profits of retailer I and retailer II are \( \pi_{r_1} \) and \( \pi_{r_2} \), respectively. When \( \Delta q' \) is obtained, the maximum trading price that buyer (retailer II) can accept is
\[
\tilde{c}_{max} = (r - v) - \frac{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'}{\frac{\tilde{Q}_1 + \tilde{q}_1}{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'}} < \tilde{c}_{r_2}^*
\]  
Similarly, the minimum trading price that the seller (retailer I) can offer is
\[
\tilde{c}_{min} = (r - v) + \frac{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'}{\frac{\tilde{Q}_1 + \tilde{q}_1}{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'}} > \tilde{c}_{r_1}^*
\]  
To achieve a deal, the condition \( \tilde{c}_{min} \leq \tilde{c}_{max} \) should hold, from which we have
\[
\tilde{c}_{max} + \tilde{c}_{min} \leq (r - v) \left[ \frac{1}{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'} \int_{\tilde{Q}_1 + \tilde{q}_1 - \Delta q'}^{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'} F_{2i}^{\ell_2} (\tilde{D}_1) d\tilde{D}_1 - \frac{1}{\tilde{Q}_1 + \tilde{q}_1} \int_{\tilde{Q}_1 + \tilde{q}_1}^{\tilde{Q}_1 + \tilde{q}_1 + \Delta q'} F_{2i}^{\ell_1} (\tilde{D}_2) d\tilde{D}_2 \right]
\]  
Under condition in equation (27), the trading at quantity \( \Delta q^* \) is Pareto-improvement for both of the two retailers.

Denote \( \Delta E \pi_{n, j} = \Delta E \pi_{nj} (\tilde{c}, \Delta q') - (\tilde{c}_{max} + \tilde{c}_{min}) \), the dynamic game with time dependence here is actually a game that: with equal probability, one of the two retailers proposes an
offer with parameter \( c \in [c_{min}, c_{max}] \subset [c_1, c_2] \), thus leads into a corresponding way for splitting \( \Delta E_\pi \) between them; if they agree, each receives its agreed share. If they fail to agree, both of them will receive zero (in this case they will receive their own backup options' expected surplus profits \( \pi_1 \) and \( \pi_2 \), respectively).

It is well known that the above model is a typical Rubinstein’s model. Therefore, a unique trading price \( c \in [c_{min}, c_{max}] \) can be found (Binmore et al., 1986).

5 Conclusions

This paper extends traditional supply chain contract with options to a new strategic model in order to achieve the maximum surplus profit of individual participant, which leads into a result: options trading provides positive effects for all participants in a supply chain. Models for cooperative and non-cooperative scenarios are established, respectively. It is proved that a cooperatively bargaining game can be realised when both of the two retailers seek for the maximum of joint surplus profits. Thus, the trading quantity can be determined and price of the trading options can be chosen by a negotiation that is dependent upon the two players’ bargaining powers. The non-cooperative model, in which retailers only concern their own profits, concludes that the latter scenario has the same strategic optimal solutions as the cooperative scenario. This result indicates that when the two retailers reach the unique optimal points for their own profits, they will also have their joint surplus profit maximised.

A further study is to analyse how to integrate channels’ coordination or supply chain performances into the proposed models by using trading options.

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